THE ENERGY DISTRIBUTION OF ATOMS IN THE FIELD OF THERMAL RADIATION

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Abstract

Using the principle of detailed balance and the assumption on the absorption cross-section consistent with available astrophysical data, we obtain the energy distribution of atoms in the field of thermal blackbody radiation and show that this distribution diverges from the Boltzmann law. There is an inversion of the high energy level population at sufficiently high temperatures.

In the unified model of compact radio sources, radio emission from active galactic nuclei and pulsars is treated as thermal radiation from an accretion disk amplified by a maser mechanism [1]. A maser amplification of thermal radio emission in continuum produces the high brightness temperatures of compact radio sources and a rapid variability of total and polarized flux density, that is characteristic for non-saturated maser sources. In particular, pulsars signals show a variability on every observable time scale up to nanoseconds [2,3]. The brightness temperatures of OH masers have the magnitude $T_b < 10^{12} K$, and those of water masers have the magnitude $T_b < 10^{15} K$ [4]. Compact extragalactic sources (active galactic nuclei) exhibit brightness temperatures in the range of $10^{10} K$ to $10^{12} K$ [5,6], so these temperatures have an order of magnitude of those of OH masers.

Maser amplification in continuum is closely connected with the stimulated origin of thermal radio emission. The induced origin of thermal radio emission follows from the relations between Einstein's coefficients for spontaneous and induced emission of radiation. However, the detailed mechanism of maser amplification has been unknown so far. In this paper we show that there is an inversion of the high energy level population at sufficiently high temperatures which can give rise to the maser amplification in continuum in a hot plasma.

The original Einstein theory of spontaneous and induced emission of radiation did not take into account a line width [7]. Here we show, in particular, that the account for a natural line width essentially changes the conclusions on the wavelength dependence of the absorption and stimulated emission cross-sections as well as the relations between Einstein's coefficients.

Consider two level system (atom) with energy levels E_1 and $E_2 > E_1$ which is in equilibrium with thermal blackbody radiation. We denote as $\nu_s = A_{21}$ the number of transitions from the upper energy level to the lower one per unit time caused by spontaneous emission of radiation with the frequency $\omega = (E_2 - E_1)/\hbar$, where $\hbar$ is the Planck constant. The number of transitions from the energy level E_2 to the energy level E_1 per unit time caused by stimulated emission of radiation may be written in the form

$$\nu_i = B_{21} B_\nu \Delta\nu = \sigma_{21} \frac{\pi B_\nu}{\hbar\omega} \Delta\nu, \tag{1}$$

where

$$B_\nu = \hbar\omega^3 / \left(2\pi^2 c^2\right) \left(exp\left(\hbar\omega/kT\right) - 1\right) \tag{2}$$

is a blackbody emissivity (Planck's function), σ_{21} is the stimulated emission cross-section, T is the temperature, k is the Boltzmann constant, c is the speed of light, and $\Delta\nu$ is the line width.

The number of transitions from the lower level to the upper one per unit time may be written in the form

$$\nu_{12} = B_{12} B_\nu \Delta\nu = \sigma_{12} \frac{\pi B_\nu}{\hbar\omega} \Delta\nu. \tag{3}$$

where σ_{12} is the absorption cross-section.

Here A_{21}, B_{21} and B_{12} are the coefficients introduced by Einstein [7], the coefficients B_{12} and B_{21} being modified to account for the line width $\Delta\nu$.

We denote as N_1 and N_2 the number of atoms occupying the energy levels E_1 and E_2, respectively. The levels E_1 and E_2 are suggested for simplicity to be non-degenerated.

In the equilibrium state the full number of transitions from the lower level to the upper one is equal to the number of reverse transitions :

$$N_1 \nu_{12} = N_1 B_{12} B_\nu \Delta\nu = N_2 \nu_{21} = N_2 \left(A_{21} + B_{21} B_\nu \Delta\nu\right). \tag{4}$$

The line width $\Delta\nu$ is suggested to be equal to the natural line width [8,9]:

$$\Delta\nu = A_{21} + B_{21} B_\nu \Delta\nu. \tag{5}$$

It is assumed that the lower level has a zero energy width, i.e. this level is a ground state. It follows from the last equation that

$$\Delta\nu = A_{21} / \left(1 - B_{21} B_\nu\right). \tag{6}$$

Substituting this expression in the equation 4 , we obtain

$$N_2/N_1 = B_{12} B_\nu = \sigma_{12} \frac{\pi B_\nu}{\hbar\omega}. \tag{7}$$

In the limit of high temperatures $T \to \infty$, corresponding to the range of frequencies $\hbar\omega < kT$, the function $B_\nu\left(T\right)$ is given by the Rayleigh-Jeans formula

$$B_\nu = 2kT\nu^2/c^2, \tag{8}$$

where ν is the frequency of radiation , $\nu = \omega/2\pi$.

Since $B_\nu \to \infty$ when $T \to \infty$, it follows from the equation (6) that the coefficient B_{21} is depending on the temperature T in such a manner that $B_{21}B_\nu < 1$. It is clear that $B_{21}B_\nu \to 1$ when $T \to \infty$. The ratio of frequencies of transitions caused by spontaneous and induced emission of radiation is given by the expression

$$\nu_s/\nu_i = A_{21}/(B_{21}B_\nu\Delta\nu) = (1 - B_{21}B_\nu)/(B_{21}B_\nu) \quad . \tag{9}$$

This ratio is approaching zero when $T \to \infty$, since $B_{21}B_\nu \to 1$. It means that in the range of frequencies $\hbar\omega < kT$ thermal radiation is produced by the stimulated emission, whereas the contribution of the spontaneous emission may be neglected .

It follows from equations (7) and (8) that in the Rayleigh-Jeans range of frequencies the ratio N_2/N_1 is given by the formula

$$N_2/N_1 = \sigma_{12}\omega kT/\left(2\pi\hbar c^2\right). \tag{10}$$

An analysis of observational data on thermal radio emission from various astrophysical objects suggests that the absorption cross-section does not depend on a wavelength of radiation and has an order of magnitude of an atomic cross-section, $\sigma_{12} \approx 10^{-15}cm^2$. Only this assumption is consistent with the observed spectra of thermal radio emission from major planets, the spectral indices of radio emission from galactic and extragalactic sources, and the wavelength dependence of radio source size.

If the absorption cross-section is constant then the energy distribution of atoms has a form

$$N_2/N_1 = const\,(E_2 - E_1)\,kT. \tag{11}$$

For all real values of the temperature and wavelength of radiation the ratio (11) is much smaller than unit. For example, at $\lambda = 1m$ the ratio (11) is less than 1, if $T < 10^{18}K$. It implies that $N_1 \approx N$, where N is the total number of atoms. There are no reasons to expect that the Boltzmann distribution is still valid for an ensemble of atoms interacting with thermal radiation.

From the relation $B_{21}B_\nu \approx 1$ we obtain the stimulated emission cross-section in the form

$$\sigma_{21} \approx \hbar\omega/(\pi B_\nu) = \lambda l_T/(2\pi), \tag{12}$$

where λ is the wavelength, and $l_T = 2\pi\hbar c/(kT)$. Thus, Einstein's relation $B_{12} = B_{21}$, equivalent to $\sigma_{12} = \sigma_{21}$, is not valid in the Rayleigh-Jeans region.

Consider now the Wien region $\hbar\omega > kT$. There are strong theoretical and observational indications of the stimulated character of thermal blackbody radiation in the whole range of spectrum. First, the spectral energy density of thermal blackbody radiation is described by a single Planck's function at all frequencies. Second, there are clear observational indications of the existence of thermal harmonics in stellar spectra and of laser type sources. The latter are connected with the induced origin of thermal radiation, similarly to well known maser sources. In particular, a possible non-saturated X-ray laser source emitting in the Fe K_α line at 6.49 keV was recently discovered in the radio-loud quasar MG J0414+0534 [10].

Assuming the stimulated origin of thermal blackbody radiation, i.e. $\nu_s < \nu_i$, from equation (8) one can obtain the relation $B_{21}B_\nu = \sigma_{21}\pi B_\nu/(\hbar\omega) \approx 1$, and then the stimulated emission cross-section in the form

$$\sigma_{21} \approx \left(\lambda^2/2\pi\right) exp\left(\hbar\omega/kT\right). \tag{13}$$

The absorption cross-section is likely to be constant in the whole range of spectrum, so the ratio N_2/N_1, according to equation (6), is given by the formula

$$N_2/N_1 \approx \left(\sigma_{12}\omega^2/2\pi c^2\right) exp\left(-\hbar\omega/kT\right), \tag{14}$$

which is somewhat similar to the Boltzmann law.

The exact formula for the ratio N_2/N_1 can be obtained from (6) :

$$N_2/N_1 = \sigma_{12}\omega^2/\left(2\pi c^2\right)\left(exp\left(\hbar\omega/kT\right) - 1\right). \tag{15}$$

The function (15) has a maximum at $\hbar\omega = 1.6kT$. It means that, in the field of thermal blackbody radiation, the excited levels with $E_2 - E_1 \approx kT$ are the most populated. Numerically, $(N_2/N_1)_{max} = 0.8 \times 10^{-15}T^2$, so the ratio N_2/N_1 is less than 1, if only $T < 3 \times 10^7 K$. For the temperatures $T > 3 \times 10^7 K$ the population of the levels corresponding to the maximum of the function (15) is inverse. It suggests that laser type sources are more easily

realized in X-ray and gamma-ray range of spectrum than in the optical region. The well known example is a possible gamma-laser in the Galactic Center emitting in the line 0.511 MeV [11].

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